Assignment 9

1. Using Euler's method, approximate $y(1)$ and $z(1)$ with h = 0.5 and again with h = 0.25 for the initial-value problem defined by

$$\begin{aligned} y^{(1)}(t) &= 2y(t) + z(t) + t - 1 \\ z^{(1)}(t) &= y(t) - 2z(t) - t - 2 \\ y(0) &= 1 \\ z(0) &= 2 \end{aligned}$$

2. In Question 1, approximate $y(0.1)$ and $z(0.1)$ using one step of Euler's method.

3. In Question 1, approximate $y(0.1)$ and $z(0.1)$ using one step of Heun's method.

4. In Question 1, approximate $y(0.1)$ and $z(0.1)$ using one step of the 4th-order Runge-Kutta method.

5. Given the results in Questions 2 and 3, what is the value of a for the adaptive Euler-Heun method for this one step if ε_{abs} = 0.1? What would be the next value of h used, and would you be recalculating the first point, or would you be calculating the value at t = 0.1 + h with the new value of h?

6. Convert the following 3rd-order initial-value problem into a system of 1st-order initial value problems:

$$\begin{aligned} y^{(3)}(t) &= 2y(t) + y^{(1)}(t) + 0.5y^{(2)}(t) + t - 1 \\ y(0) &= 1.2 \\ y^{(1)}(0) &= 1.3 \\ y^{(2)}(0) &= 1.4 \end{aligned}$$

7. Convert the following system of three 2rd-order initial-value problem into a system of 1st-order initial value problems.

$$\begin{aligned} x^{(2)}(t) &= 2x(t)y^{(1)}(t) - 1.2 \\ y^{(2)}(t) &= 4y(t)z^{(1)}(t) - 1.7 \\ z^{(2)}(t) &= 3z(t)x^{(1)}(t) - 1.9 \\ x(0) &= 1.4 \\ x^{(1)}(0) &= -1.5 \\ y(0) &= 1.6 \\ y^{(1)}(0) &= -1.9 \\ z(0) &= 0.1 \\ z^{(1)}(0) &= -1.3 \end{aligned}$$

8. Use three steps of the shooting method to approximate a solution to the boundary-value problem defined by

$$u^{(2)}(x) = 0.3u^{(1)}(x) + 0.1u(x) - x - 0.2$$
$$u(0) = 2$$
$$u(1) = 3$$

At each step of the shooting method, you will use Euler's method with $h = 0.2$.

9. The actual solution to the boundary-value problem given in Question 8 is

$$u(x) = 3\frac{e^{-0.2x}\left(10e^{0.5} - 7\right) - e^{0.5x}\left(10e^{-0.2} - 7\right)}{e^{0.5} - e^{-0.2}} + 10x - 28.$$

How close is your approximation to this solution at $x = 0.2, 0.4, 0.6$ and 0.8?